Roll No. ....

41252

B. Sc. (Hons.) Maths 4th Semester Examination – May, 2019

SPECIAL FUNCTIONS AND INTEGRAL TRANSFORMS

Paper : BH242

Time : Three hours] ***[Maximum Marks : 60***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. **9** (Unit – **V**) is ***compulsory***.

UNIT – I

1. (a) Find the power series solution of

$$(x^2-1)\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0,\ y(0)=2,\ y'(0)=3$$

(b) Solve :

$$x^2\frac{d^2y}{dx^2}+(x+x^2)\frac{dy}{dx}+(x-a)y=0$$

P. T. O.

2. (a) Show that :

$$\frac{d}{dx}\left[J_n^2(x)\right]=\frac{x}{2n}\left[J_{n-1}^2(x)-J_{n+1}^2(x)\right]$$

(b) Find the solution of the following equations in terms of Bessel's function

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}+4\left(x^2-\frac{n^2}{x^2}\right)y=0.$$

UNIT– II

3. (a) Show that :

$$P_{2n}(0)=(-1)^n\frac{1.3.5......(2n-1)}{2.4.6.......2n}\frac{2n!}{(n!)^2(2^n)^2}$$

(b) Prove that :

$$\int_{-1}^{1}x\,P_n(x)\,P_{n-1}(x)\,dx=\frac{2n}{4n^2-1}$$

4. (a) To show that :

$$e^{2tx-t^2}=\sum_{n=0}^{\infty}\frac{t^n}{n!}H_n(x)$$

(b) Show that :

$$\int_{-\infty}^{\infty}e^{-x^2}[H_n(x)]^2\,dx=2^n\,n!\int_{-\infty}^{\infty}e^{-x^2}dx=2^n\,n!\sqrt{\pi}$$

(b) Find $f(x)$ if its Fourier sine transform is $\frac{s}{1+s^2}$.

(a) Using Parseval's Identity, prove that

$$\int_0^\infty \frac{x^2\,dx}{(x^2+1)^2} = \pi/4.$$

(b) Solve $\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2}$ with Boundary conditions.

(i) $u = u_0$ when $x = 0$, $t > 0$ and the initial condition

(ii) $u = 0$ when $t = 0$, $x > 0$

UNIT– V

Find the radius of convergence

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{5m}(x+1)^{3/m}$$

Show that $J_0''(x) = \frac{1}{x}J_1(x) - J_0(x)$

Find the value of $H_{2n}(0)$.

Find $L\left[\sin\frac{t}{2}\sin\frac{3t}{2}\right]$

$L^{-1}\left[\frac{s}{4s^2+15}\right]$

Find Fourier cosine transform of e^{-5x}.